Is the Market Truly in a Random Walk? Searching for the efficient market hypothesis with an AI assistant economist

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Abstract

The equity market is known for its uncertainty and randomness. While the market and the participating traders' may seem like independent entities in their own right, but it is the foray of traders' that makes the market in a random walk, as the market's volatility influences the traders' judgement on which action to take; the market and traders are "entangled together" in this way. This paper presents a methodology to model both the market's volatility and traders' actions by drawing off the concept of quantum superposition to illustrate that it is indeed the "interactions" of both the market and traders that results in the random walk, fully conforming to the efficient market hypothesis. We've also developed an AI assistant economist that's powered by a quantum-like evolutionary algorithm to produce short horizon predictions of the future trend of the market based on Darwinian natural selection.

Keywords: random walk, efficient market hypothesis, genetic programming, machine learning, AI assistant economist, quantum-like evolutionary algorithm

1 Introduction

The stock market has been known to be a volatile place [1], and there have been many theories that attempt to formulate the inner workings of the market's regularities [2-4]. In 1900, doctoral student Louis Bachelier, who had a strong interest in modeling the price fluctuations of the Paris Stock Exchange wrote his thesis *The Theory of Speculation* which formulated the first mathematical theory of the stochastic processes of the market [5]. Bachelier argued, based on his observations, that the price fluctuations of stocks act in the same way of the atoms fluctuate around randomly in physical phenomena like Brownian motion [6], thus prices take "random walks" around their true values where no one can accurately predict by any means possible.

In 1965, Eugene Fama developed what is now known as the efficient market hypothesis (EMH), which in a crash course lesson basically states that financial markets are "informatively efficient" and that no one can accurately predict the market's future by any means possible [7]. Essentially EMH is a cornerstone of modern-day financial theory not because it is complicated or provides a bleak outlook on the market but because it complements the fact that the market is indeed in a random walk, and that the current price of stocks reflects all the available information, the distant values of the historical prices and outlying to-happen future values don't have much of an effect on the price right now.

In 1973, Fisher Black, Robert Morton, and Myron Scholes developed a model named after them, the BSM model, one that attempted to describe the market's behavior in pure mathematical terms [8]. Their model, in which the equation of the same name can be derived of, provides a theoretical framework to derive a price estimate of European call and put options. Briefly the Black-Scholes model uses the log-normal distribution probabilities to account for volatility of the underlying asset when calculating the price of an option on the return an investor gets less than the amount they have to pay. The log-normal distribution of the returns calculated in the model is based on theories of Brownian motion, also stating that asset prices exhibit similar behavior to natural organic Brownian motion movement.

The mainstream ways of describing the market have been to model the state of the market and the traders' actions separately, with the market being treated as a physical particle or mathematical entity, and the traders' being seen as a completely separate external factor that should be left alone when attempting to describe the market.

To model the behavior of traders', Amos Tversky and Daniel Kahneman formulated prospect theory [9-10]. Their theory attempts to factor in the "humanity" aspect of trading, arguing that traders value gains and losses differently. Also known as loss aversion theory, Tversky and Kahneman put forth that when presented with two different options where both are equal but one is presented as riskier, most people will tend to pick the less risky one. They proposed that losses will always carry a greater emotional impact on an individual, thus gains are generally perceived as greater probability wise.

When studying the market, it is crucial to factor in both the market itself and the participating traders involved and not separately, because it is together, both the market and traders' overall is what makes up the movement of the market; it is the market's volatility that hampers the participating traders' decisive decision-making ability (traders are initially hesitant whether to buy or sell) and in turn it is the "collective effort" of all the traders' (some will buy while others will sell) that eventually determine the market's trend direction (increase or decrease). Thus, the market and traders are intertwined or "entangled" with each other, the market's volatility affects the traders' and the traders' actions in turn determines the trend of the market and vice versa, in this continuous looping cycle which fully reflects that the market is truly in a random walk.

We present a methodology to model both the market's volatile movement (increase or decrease) and the participating trader's actions (buy or sell) by utilizing the concept of the quantum principle of superposition and illustrate that it is the two "entangled" that causes the market to be in a random walk which fully conforms to the observed market movement as stated by the efficient market hypothesis [11]. Building off of our methodology, we've developed an AI assistant economist powered by our quantum-like evolutionary algorithm that can produce a short horizon prediction (one week) of the market's future movement by studying one month of data from the Dow Jones Index [12].

The contributions of this paper are: 1) we model both the market's movement and traders' actions by utilizing the concept of quantum superposition and show that they are intertwined which is what causes the market to be in a random walk, and 2) we've developed a quantum-like evolutionary algorithm that powers an AI assistant economist to produce short horizon forecasts of the market's future trend (increase or decrease).

The rest of the paper is structured as follows: Section 2 details the methodology. Section 3 are the results. Section 4 is the conclusion.

2 Methods

The volatility of the market and the hesitation of the traders' actions are essentially intertwined; the uncertain nature of the market hampers traders' decision-making ability of when to buy and sell, and in turn it is the "collective" actions of all the participating traders' that determines the markets' closing price in this ever-changing cycle between the market and traders.

To effectively model both the volatility of the market and the traders' actions of buy and sell, the concept of quantum superposition principle [13-15] can be utilized; by "superposing" both the market's states and the traders' actions. This can be modeled as in (1) and (2).

$$|\mathrm{Q}\rangle = \mathrm{c}_1|\mathrm{q}_1\rangle + \mathrm{c}_2|\mathrm{q}_2\rangle \tag{1}$$

Where $|\mathrm{q}_1\rangle$ denoting the market increases; $|\mathrm{q}_2\rangle$ denoting the market decreases. $\omega_1 = |\mathrm{c}_1|^2$ is the objective frequency that the market increases; $\omega_2 = |\mathrm{c}_2|^2$ is the objective frequency that the market decreases.

$$|A\rangle = \mu_1|a_1\rangle + \mu_2|a_2\rangle \tag{2}$$

Where $|a_1\rangle$ denotes the trader believes that the market increases; $|a_2\rangle$ denotes the trader believes that the market decreases. $p_1 = |\mu_1|^2$ are the trader's degree of beliefs that the market increases; $p_2 = |\mu_2|^2$ are the trader's degree of beliefs that the market decreases.

The market and all the participating traders can be described as a complex system, as (3).

$$|\psi\rangle = c_1|q_1\rangle \otimes \prod_{i=1}^{N}|a_1^i\rangle + c_2|q_2\rangle \otimes \prod_{i=1}^{N}|a_2^i\rangle \tag{3}$$

Where N is the number of traders in the group. The density operator of the complex system can be described as (4).

$$\rho_{\text{market+traders}} = |\psi\rangle\langle\psi| = \omega_1|q_1\rangle\langle q_1| + \omega_2|q_2\rangle\langle q_2| + \left[c_1c_2^*|q_2\rangle\langle q_1| \otimes \prod_{i=1}^{N}\langle a_1^i|a_2^i\rangle + \text{H.C.}\right] \tag{4}$$

Where the third term is a non-diagonalization term that represents the superposition of the market either increasing or decreasing as well as the traders' being unable to deduce whether the market will increase or decrease. Traders' will tend to randomly "guess" that the market is increasing or decreasing; the traders' believing whether the market is increasing or decreasing is "orthogonal", and when the number of participating traders' is very large then the expectations of all the traders' for whether the market will increase or decrease are then zero as (5).

$$\prod_{i=1}^{N}\langle a_1^i|a_2^i\rangle \xrightarrow{N\to\infty} 0 \tag{5}$$

(4) then becomes (6).

$$\rho_{\text{market+traders}} \xrightarrow{N\to\infty} \omega_1|q_1\rangle\langle q_1| + \omega_2|q_2\rangle\langle q_2| \tag{6}$$

When there is a vast number of participating traders involved ($N \to \infty$), the market and all the participating traders as a whole tends to be in a random walk ($\rho_{\text{market+traders}} \approx \rho_{\text{market}}$) as outlined by the efficient market hypothesis. ρ_{market} as (7) is the actual observed density operator of the market; where ω_1 is the observed objective frequency that the market will increase and ω_2 is the observed objective frequency that the market will decrease ($\omega_1 \approx \omega_2 = 0.5$).

$$\rho_{\text{market}} = \omega_1|q_1\rangle\langle q_1| + \omega_2|q_2\rangle\langle q_2| \tag{7}$$

We have shown above that it is the market and the participating traders as a collective whole that makes the market become in a random walk. It is widely acknowledged that nobody can accurately predict the future trend of the market in the long run. Now the question becomes: is it possible to produce a short horizon forecast of the market's future trend?

To answer this question, we've developed a quantum-like evolutionary algorithm to power the AI assistant economist that utilizes both the quantum superposition principle and Genetic Programming (GP) [16-18] to produce possible short horizon predictions of the market's future trend by machine learning historical trading data.

For the AI assistant economist (AI agent), we can hypothesize that before the AI agent makes its decision, "believes" whether the market will increase or decrease, are "superposed simultaneously" in its "mind", which can be described by the density operator as in (8).

$$\rho_{\text{agent}} = |A\rangle\langle A| = p_1|a_1\rangle\langle a_1| + p_2|a_2\rangle\langle a_2| + \mu_1\mu_2^*|a_1\rangle\langle a_2| + \mu_1^*\mu_2|a_2\rangle\langle a_1| \tag{8}$$

Where p_1 are the AI agent's degree of beliefs that the market increases, p_2 are the AI agent's degree of beliefs that the market decreases. The third and fourth terms in (8) are the “quantum interference” terms that indicate the AI agent’s “mind” is undecided on whether the market will increase or decrease, where the AI agent can “think” that the market is both increasing and decreasing.

When an AI agent actually “decides” on whether the market increases or decreases, a projection of pure state to mixed state happens in the AI agent’s “mind” as (9) which describes the decision-making process of an AI agent.

$$\rho_{\text{agent}} \xrightarrow{\text{Decide}} \rho'_{\text{agent}} = p_1|a_1\rangle\langle a_1| + p_2|a_2\rangle\langle a_2| \tag{9}$$

The decision-making process is essentially just a projection from pure state to mixed state, where GP, an algorithm based off of Darwinian natural selection [19], is utilized to evolve a satisfactory pure state. The pure state is essentially just a 2x2 matrix, where (9) can be described by the matrix form represented in (10).

$$\rho_{\text{agent}} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \xrightarrow{\text{projection}} \rho'_{\text{agent}} = \begin{bmatrix} p_1 & 0 \\ 0 & p_2 \end{bmatrix} = p_1|a_1\rangle\langle a_1| + p_2|a_2\rangle\langle a_2| \tag{10a}$$

$$|a_1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |a_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix};\ |a_1\rangle\langle a_1| = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, |a_2\rangle\langle a_2| = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{10b}$$

Because the pure density operator ρ_{agent} is just an arbitrary 2x2 matrix, we can then approximately construct this density operator with the 8 most basic quantum gates as (11) leading it to become a "matrix tree" [20].

$$\left\{ \begin{matrix} H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \\ S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix} \right\} \tag{11}$$

After constructing an individual "matrix tree", we can then construct a population of "matrix trees", and then by using the fitness function as the evaluation criteria, the most satisfactory density matrix ρ_{agent} from the population is evolved through generations of natural selection. The “matrix tree” is essentially a decision tree that guides the AI agent which strategies to “take” with corresponding actions. At any given time, the expected value under the current environment (the market is increasing or decreasing) and the corresponding actions (the AI agent “thinks” that the market is increasing or decreasing) can be represented as (12).

$$\rho_{\text{market}} \otimes \rho_{\text{agent}} = \omega_1 p_1 |\langle q_1||a_1\rangle|^2 + \omega_1 p_2 |\langle q_1||a_2\rangle|^2 + \omega_2 p_1 |\langle q_2||a_1\rangle|^2 + \omega_2 p_2 |\langle q_2||a_2\rangle|^2 \tag{12}$$

Where (12) is the composite system of the market and the AI agent. Essentially (12) describes the four possible outcomes of every “decision” made by the AI agent; if the market is increasing or decreasing and the AI agent “thinks” or “doesn’t think so” and vice versa; when the AI agent “thinks” correctly in line with the corresponding motion of the market it’s “rewarded”, if not it’s “punished”. The expected value for the AI agent is the possible scenarios of what the outcome could be paired with the state of the market that’s being observed, as in (13). If the training data has N number of values, then the fitness function for the "matrix tree" is defined as (14), and it is the total sum of all the expected values of each "decision made" by the AI agent.

$$EV_t = \begin{cases} \omega_1 p_1, \text{market increases and AI agent "thinks" so with probaility } p_1 \\ -\omega_1 p_2, \text{market increases and AI agent doesn't "think" so with probability } p_2 \\ -\omega_2 p_1, \text{market decreases and AI agent doesn't "think" so with probability } p_1 \\ \omega_2 p_2, \text{market decreaes and AI agent "thinks" so with probability } p_2 \end{cases} \tag{13}$$

$$\text{fitness}_{\text{matrixTree}} = \sum_{t=1}^{N} EV_t \tag{14}$$

If there are M number of individuals in a population of "matrix trees", the most satisfactory “matrix tree” is the one that possess the maximum fitness function that can be described as in (15).

$$\rho_{agent}^{output} = \arg\max_{a}\{fitness_{matrixTree}, k = 1, \cdots, M\} \quad (15)$$

By learning historical data, the more rewards that are reaped then the more accurate chance there is of predicting the next outcome of whether the market will increase or decrease.

3 Results

In this paper we produced short horizon forecast outcomes by studying a small sample of data. The data used was from October 4th, 2024 to November 1st, 2024 of the Dow Jones Industrial Average Index. The data was trained twice consecutively, with 6 possible forecast outcomes produced each session by 3 AI agents. From the 12 total possible forecast outcomes produced, by means of majority rules, a final trend sequence of whether the market will increase or decrease is produced to analyze the future trend of the market. The fitting results of the two training sessions are shown in Figure 1 and Figure 2. The trend sequence produced by means of majority rules from the 12 individual possible forecast outcomes is shown in Table 1.

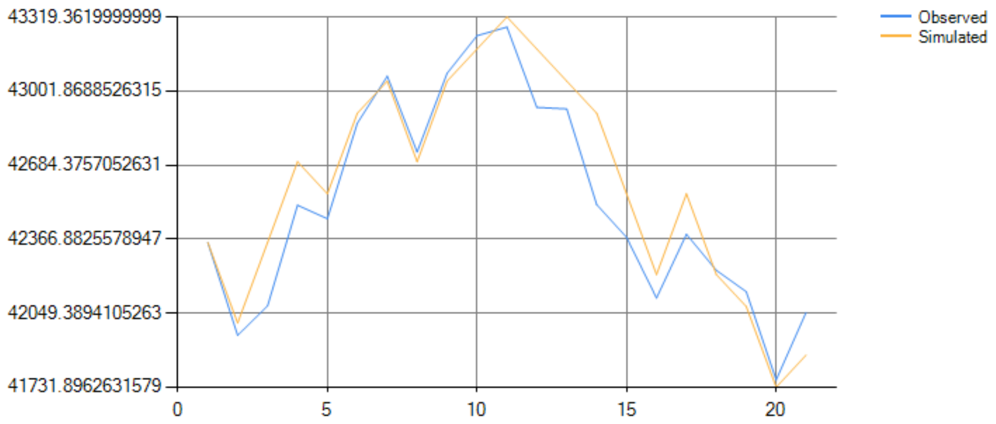

Fig. 1 The fitting results of the first training session.

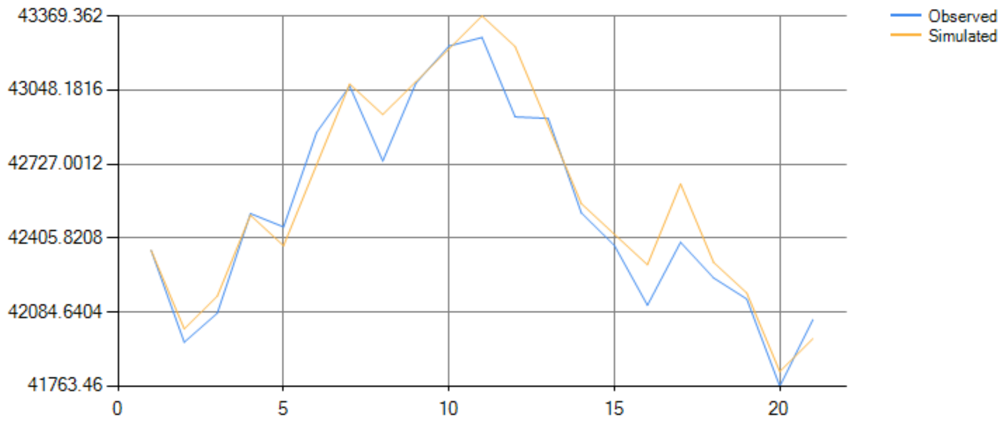

Fig. 2 The fitting results of the second training session.

Table 1 Final action sequence produced by logic tree

Date	DJIA Trend	Trend Sequence
11/04/2024	Decreased	0
11/05/2024	Increased	0
11/06/2024	Increased	0
11/07/2024	Decreased	1
11/08/2024	Increased	0

Using this action sequence produced, the future trend of the market and the market's volatility can be analyzed. For the 12 individual forecast outcomes that comprise of how this final action sequence was produced, please refer to the supplementary materials. The actual recorded tend of the Dow Jones for the following week is listed in the DJIA Trend column, while the trend sequence that's produced is listed in the trend sequence column where 0 represents the AI agent "believes" that the market will increase and 1 represents the AI agent "believes" that the market will decrease.

For this specific action sequence produced, the predicted trend of the market will be {Increase, Increase, Increase, Decrease, Increase}, in which only the first value predicted was wrong, thus resulting in odds of 80% accuracy. In this particular case, even though the odds reached 80% accuracy, however if the number of possible forecast outcomes produced were increased, or if the forecast horizon was extended, then the odds may have been lower closer to 50-50, which is exactly in line with what the efficient market hypothesis states that the market is indeed in a random walk.

4 Conclusion

In this paper, we presented a methodology to describe both the volatility of the market and the participating traders' actions together in an intertwined model. By utilizing the concept of quantum superposition principle to model both the state of the market and all the participating traders' possible actions as a whole, we show that the market is indeed in a random walk as stated by the efficient method hypothesis. Unlike compared to traditional methods that don't factor in the participating traders which the treat the market as a mathematical or physical entity (particle) where calculus is needed to describe it; we are able to subtly model both the market without a statistical approach as well as taking into account the "humanity" aspect of the participating traders' involved by using the quantum superposition concept to "superpose" all the possibilities. We also show that through the cooperation of 3 AI agents, "they" are able to produce a single forecast with 80% odds by majority rules.

Future research will include producing a larger forecast sample for a short forecast horizon and producing a trend sequence for a larger forecast horizon to see whether if more forecasts are produced and the horizon is expanded the odds of the market trend increasing or decreasing will gravitate to 50-50, and no matter how hard we try the future of the market is unpredictable, as stated by the efficient market hypothesis.

Declarations

Data availability. The authors confirm that all data that support the findings of this study are included within the main text and its supplementary materials. All data are also available from the corresponding author K.X., upon reasonable request.

Authors' contributions. All authors conducted the research and contributed to the development of the model. All authors reviewed the manuscript.

Conflict of interests. The authors declare that they have no competing interests.

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References

[1] Shiller, Robert J. Market volatility. Cambridge, Mass: MIT Press, 1991.

[2] Knight, F. Risk, Uncertainty, and Profit. New York: Houghton Mifflin Company, 1921.

[3] Baaquie, B. E. Quantum Finance: Path integrals and Hamiltonians for options and interest rates. Cambridge: Cambridge University Press, 2007.

[4] Fox, Justin. The myth of the rational market a history of risk, reward, and delusion on wall street. S.l.: HarperCollins, 2009.

[5] Bachelier, Louis, Mark H. A. Davis, and Alison Etheridge. Theory of speculation: The Origins of Modern Finance. Princeton, N.J: Princeton University Press, 2006.

[6] Broadbent, Thomas Andrew, Albert B. Einstein, H. Hertz, Hugh L. Dryden, F. P. Murnaghan and Harold Bateman. “Investigations on the Theory of the Brownian Movement.” *The Mathematical Gazette* 41 (1928): 231 – 231.

[7] Fama, Eugene F. “Efficient Capital Markets: A Review of Theory and Empirical Work.” The Journal of Finance 25, no. 2 (1970): 383 – 417.

[8] Black, Fischer, and Myron Scholes. “The Pricing of Options and Corporate Liabilities.” Journal of Political Economy 81, no. 3 (1973): 637 – 54.

[9] Kahneman, D. and Tversky, A. (1979) Prospect Theory: An Analysis of Decision under Risk. Econometrica: Journal of the Econometric Society, 47, 263-291.

[10] Kahneman, Daniel. Thinking, fast and slow. New York: Farrar, Straus and Giroux - Macmillan, 2013.

[11] Xin, Houwen, Xin, Lizhi. “Liangzi Jinrong: Buquedingxing Shichang Yuanli, Jizhi He Suanfa = Quantum Finance: Principle, mechanism and algorithm of uncertain market.” Hefei: University of Science and Technology of China Press, 2021.

[12] Xin, Kevin and Lizhi Xin. “QxEAI: Assisting equity trading with quantum-like evolutionary algorithm.” (2024).

[13] Silverman, Mark P. Quantum Superposition: Counterintuitive consequences of coherence, entanglement, and interference. Berlin: Springer, 2008.

[14] Feynman, Richard P. The Feynman Lectures on Physics. Volume III, Quantum Mechanics. Basic Books, 2015.

[15] Dirac, P.A.M. The Principles of Quantum Mechanics; Oxford University Press: Oxford, UK, 1958.

[16] Holland, J. Adaptation in Natural and Artificial System; University of Michigan Press: Ann Arbor, MI, USA, 1975.

[17] Koza, J.R. Genetic Programming, on the Programming of Computers by Means of Natural Selection; MIT Press: Cambridge, MA, USA, 1992.

[18] Koza, J.R. Genetic Programming II, Automatic Discovery of Reusable Programs; MIT Press: Cambridge, MA, USA, 1994.

[19] Darwin, Charles, 1809-1882. On the Origin of Species by Means of Natural Selection, or Preservation of Favoured Races in the Struggle for Life. London: John Murray, 1859.

[20] Xin, L.; Xin, K.; Xin, H. On Laws of Thought—A Quantum-like Machine Learning Approach. Entropy 2023, 25, 1213.